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"Black-Scholes-Artificial Neural Network": A novel option pricing model

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Abstract
Purpose: This study conducts a comparative study of various options pricing models and introduces a new model.
Research methodology: This paper reviews eight option pricing models, including the Black-Scholes-Merton model (BSM), Monte Carlo simulation (MC), Heston, GARCH, Lattice, Jump Diffusion models (JDM), Normal Inverse Gaussian-Cox-Ingersoll-Ross Model, and a novel model called Black-Scholes-Artificial Neural Network (BSANN). The objective is to predict the European call and put options using a payoff calculation. The underlying asset is Khodro, a famous automobile producer company in Iran, for the last year. The daily prices were also used as historical data. The primary software used for the calculations and plots was MATLAB. An Excel option pricing toolbox was used to obtain more accurate and improved results.
Results: Based on the results, it can be concluded that the proposed model, BS-ANN, provides the most accurate estimation with the lowest standard deviation.
Limitations: There are several limitations to be considered when choosing an underlying asset. An important factor is the availability of sufficient data on the number of shared transactions. Another limitation of this study is the absence of trading halts. Additionally, caution is crucial when selecting an appropriate number of estimated parameters.
Contribution: By utilizing the presented model, researchers, individuals, investors, and stock market analysts interested in trading can enhance their estimations.
Novelty: The most significant novelty of this study is the presentation of a hybrid model incorporating unique features.
Keywords: *Hybrid Option Pricing Models, Artificial Neural Network, Financial Engineering, Option Price Estimation*
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1. Introduction

These options have become extremely popular. There are two main reasons for this finding. First, options are interesting to investors because of their potential for speculation and hedging (Jiang & Pan, 2022). Second, there is an organized and structured way to determine their value. Thus, they can be purchased and sold confidently.

Options are securities whose values are derived from other securities, known as "underlying assets" (Parameswaran, 2022). There is a lot of data on the underlying assets. Pricing an option can be a time consuming task. Thus, it makes sense to use fast and accurate option pricing models (Almeida, Fan, Freire, & Tang, 2023). Options provide investors with a payoff that depends on the value of the underlying asset (Pucci di Benisichi 2019). The payoff is obtained by calculating the difference between the initial and future prices (Guo 2022). Therefore, we should analyze the future prices of the underlying asset with multiple possible outcomes.

Generally, there are two options: 1. Call option, 2. Put option. An option to buy a security is called a "call" option, while an option to sell is a "put" option (Guides, 2018). Many rules exist regarding how and when an option can be exercised. One of the major sorting options is to divide them into four categories: 1. European, 2. American, 3. Asian, and 4. African. Vanilla and 4. Exotic. The "European option" can be exercised on a specific future date for a price agreed upon in the contract. An "American option" can be exercised on any day prior to a specified future date. The options mentioned above are generally referred to as "vanilla options" to indicate that they are standardized and considered less complex compared to "exotic options" (Almeida et al., 2023). In Part Three, which focuses on the methodology, the subject of options is developed and expanded.

There are many studies on option-pricing models. However, in this article, we have attempted to examine a diverse range of option pricing models, showing impressive results and demonstrating the calculation of payoffs. In addition, we present a hybrid model. The main goal of this study is to introduce a hybrid model. While conducting a comparative study of different option pricing models with improved codes, the second goal is to determine the best model based on its error and accompanying plots. An important stock, Khodro, has been selected as the underlying asset. The main contribution of this article is that, on one hand, we have attempted to present a hybrid model for option pricing. However, the code has been improved. Different software programs, such as MATLAB and Excel, were used to improve the results.

Artificial intelligence (AI) has emerged as an important concept in recent studies. It has several characteristics that differentiate it from other methods, such as econometric, mathematical, and statistical models (Kase & Laka, 2019). These characteristics include the ability to accelerate calculations, compatibility with intricate data structures (El Fallahi et al., 2022), and enhancements through training and learning. Researchers are continuously striving to uncover new applications of AI in various fields, such as medicine, finance, economics, and transportation.

Consequently, we have attempted to utilize artificial neural networks (ANN) as a method for option pricing. A hybrid model known as the Black-Scholes-Artificial Neural Network (BSANN) was used to predict Khodro option prices. More details about this method are provided in the methodology section.

As is evident, like all other models or methods, options have various parameters and variables, such as strike price and expiration date (time to maturity). Therefore, if these parameters are carefully adjusted, taking into account economic factors such as the inflation rate and risk factors such as political and interest rate risks, it could be possible to make a fair and reasonable prediction.

Now that you understand the status of options compared to other investment approaches, it is time to examine the concepts in options, such as option categories and related keywords.

Table 1. derivative concepts

Row	Concepts	Definition
1	**Call Option**	Option to purchase the underlying asset
2	**Put Option**	Option to sell the underlying asset
3	**Options Contract**	The agreement between the writer and the buyer
4	**Expiration Date**	The last day on which an options contract can be exercised
5	**Strike Price**	The pre-determined price at which the underlying asset can be bought or sold
6	**Intrinsic Value**	The current value of the option's underlying asset
7	**Time Value**	The premium, or the extra amount that traders are willing to pay for an option
8	**Vanilla Option**	A standard option with no unique features, terms, or conditions

9	**American Option**	Options that can be exercised at any time before the expiration date
10	**European Option**	Options that can only be exercised at the expiration date
11	**Exotic Option**	Any option with a complex structure or payoff calculation
12	**Bermudan Option**	An option that can only be exercised during a predefined portion of its lifespan

Table 1 shows two types of options: call options and put options, along with various option categories including Vanilla, American, and European options.

The remainder of this paper is organized as follows. The second section is dedicated to a review of the literature. The third part describes the methodology and applied models, including the statistical population, period of time, and data statistics. Part Four focuses on the findings, results, and experimental processes. The final section consists of conclusions and remarks. For more information, please refer to the Appendix.

2. Literature review

There are several methods, such as mathematical models and computational algorithms, for recognizing the behavior of asset prices that offer an understanding of fairness (Das et al., 2023; Mayes and Govender, 2019; Tudor, 2022). Louis Bachelier published his doctoral thesis in 1900. In his thesis, Bachelier made the first attempt to model stock price movements as random walks. This thesis also addresses the problem of option pricing. In 1964, Samuelson modified Bachelier's model by replacing the stock price with the return in the original model. This correction eliminated the unrealistic negative value of stock prices in the original model. In 1973, Black and Scholes corrected Samuelson's model. Compared to Samuelson, the risk-free interest rate is included in the formula. In summary, the following table was created:

Table 2. Option pricing history

Period	Explanations
1900	Bachelier, the purpose of risk management
1950s	The discovery of Bachelier's work
1960s	Samuelson's formula, which contains the expected return
1967	Thorp and Kassouf beat the market by going long on stocks and shorting warrants
1973	Black-Scholes
1997	The Black-Merton-Scholes formula assumes that all investors exist in a risk-neutral world, where the expected return is equal to the risk-free interest rate

The most popular models, such as Black-Scholes-Merton and Monte Carlo simulations, use the same factors, such as time to maturity, strike price, and interest rate, in option pricing (Parameswaran, 2022). However, volatility is one of the major factors that make these models unique to each other. Volatility is a major issue in financial mathematics. First, it helps us understand price dynamics; second, it is the only factor that cannot be seen; third, volatility has always been an interesting issue in a wide range of research areas (Wang, Cheng, Yin, & Yu, 2022).

Black and Scholes (1973) presented the first complete equilibrium option-pricing model. In the same year, Merton attempted to redefine the Black and Scholes model and made some modifications. The jump-diffusion model, suggested by Merton, is a stochastic process that involves both jumps and diffusion. Merton modified the BSM to account for dividends and variable interest rates (Parameswaran, 2022). One popular model is the Monte Carlo option pricing model, which utilizes Monte Carlo simulation and processes to calculate the valuation of an option. Boyle (Trinh & Hanzon, 2022) performed the first calculation of option pricing using Monte Carlo. In finance, the lattice model is used for the valuation of options with discrete-time model assumptions. The simplest lattice model is the binomial option pricing model (BM), which was developed in 1979 by Cox-Ross-Rubinstein and is commonly referred to as the CRR model (He, Coolen, & Coolen-Maturi, 2021). Some option pricing models assume constant volatility, whereas many financial time series are characterized by timevarying volatility. The GARCH models proposed by Engle (1982), Dana (2016), and Bollerslev (1986) consider the dynamics of returns. Another model is the Levy process (Bertoin, 1992; Levy, 1992), which is a stochastic process with independent and stationary increments (Velasquez, 2020).

Every option-pricing model includes various assumptions and hypotheses. For example, the B-S model contains the assumptions considered (Karagozoglu, 2022). However, this remains a question. Are these assumptions true in real-life? Therefore, these assumptions must be modified. Owing to changes in these assumptions, different models have emerged. One of the assumptions in Black-Scholes-Merton (B-S-M) is that asset price returns follow a lognormal distribution. While most financial data exhibit skewness, kurtosis, and term structures, implied volatility is also observed (Bali et al., 2019). Therefore, some researchers recognize the volatility of asset returns as a hidden stochastic process and conclude that the B-S-M model cannot be used for stochastic volatility. As a result, they changed their methods and started using other processes, such as Levy, Monte Carlo simulation, and Fourier.

Recently, there has been considerable interest in the development of artificial neural networks (ANNs) for solving various problems (Roy, 2023). Neural networks, which are capable of learning relationships from data, represent a class of robust nonlinear models inspired by the neural architecture of the brain. Theoretical advances along with hardware and software innovations have addressed previous limitations in the implementation of machine learning, making neural network methods accessible to a diverse range of disciplines (Qin & Chiang, 2019).

Financial applications that require pattern matching, classification, and prediction, such as corporate bond ratings (Hamid & Razak, 2023), trend prediction (Farahani & Hajiagha, 2021; Taheri & Aliakbary, 2022), failure prediction (Veganzones & Severin, 2021), and underwriting (Jansen, Nguyen, & Shams, 2020), have proven to be excellent candidates for this new technology (Wani, 2022).

The following table (Table 3) displays various research studies and models of option pricing, including their methodologies and results.

Table 3. Research background

Row	Author	Title	Model/Main Variables	Statistical population	Journal	Findings
1.	Bendob and Bentouir (2019)	Options Pricing by Monte Carlo Simulation, Binomial Tree and BMS Model: a Comparative Study	Options pricing, option markets, the Black-Scholes model, the Binomial model, the Monte-Carlo simulation model, and Greek letters.	Nifty50 option index during the period of 25/07/2014 to 30/06/2016	Journal of Banking and Financial Economics	- All models are overpriced across all Moneyness categories, with a high level of volatility in the In-the-money category. -The Monte Carlo simulation method outperforms when the volatility is lower, while the Black-Scholes model and the Binomial model outperform the entire sample, regardless of the Moneyness being ignored.

2.	Rebentrost, Gupt, and Bromley (2018)	Quantum Computational Finance: Monte Carlo Pricing of Fnancial Derivatives	Black-Scholes-Merton option pricing, classical Monte Carlo pricing, Quantum algorithm for Monte Carlo	European option, Asian option	A Physical Review	- This paper demonstrated the possibility of preparing relevant probabilities in quantum superposition. -They discussed the European call option and the Asian option, both of which depend on the average asset price before the maturity date.
3	Jang, Yoon, Kim, Gu, and Kim (2021)	Deep Option: A Novel Option Pricing Framework Based on Deep Learning with Fused Distilled Data from Multiple Parametric Methods	Option pricing, Delta hedging, Deep learning, Data fusion, Data distillation	S&P 500 European call options, EuroStoxx50 call options, and Hang Seng Index put options	Information Fusion	- The proposed model outperforms parametric methods and other machine learning methods, reducing the overall mean absolute percentage error (MAPE) by more than 50%.
4	Lin, Li, Gao, and Wu (2021)	The Numerical Simulation of Quanto Option Prices Using Bayesian Statistical Methods	Quanto option, Foreign asset, Exchange rate, Correlation, Bayesian statistical inference, Markov Chain Monte Carlo	China	Physica A: Statistical Mechanics and its Applications	- This paper presents a novel application of Bayesian methods in the pricing of multi-asset options.
5	Liu and Huang (2019)	Research on Pricing of Carbon Options Based on GARCH and B-S Model	Carbon options; Carbon emission rights; GARCH model; Black-Scholes model	Daily closing price of EU emission allowance futures options	Journal of Applied Science and Engineering Innovation	- This paper discusses the pricing of carbon options in the EU carbon trading market, with the aim of providing a scientific pricing framework for future carbon options trading in China. Learn from the meaning.

6	Simiyu, Waititu, and Akinyi (2019)	A Hybrid Artificial Neural Network Model for Option Pricing	A Hybrid model, Artificial Neural Networks, option pricing time series models, ANN-GJR-GARCH model	Intraday data for the AAPL stock option for the period between December 2016 and March 2017 with 56,238 data points.	Journal of Mathematics and Statistics	- The results indicate that the hybrid model outperforms the pure ANN model not only in forecasting accuracy, but also in terms of training time and model complexity.
7	Arin and Ozbayoglu (2022)	Deep Learning Based Hybrid Computational Intelligence Models for Options Pricing	Option pricing, computational intelligence, deep neural networks, machine learning, Black Scholes	European option	Computational Economics	- The results indicate that the proposed models can generate more accurate prices for all classes of options. Compared with BS using annualized 20 intraday returns as volatility, there is a 94.5% improvement in option pricing in terms of mean squared error when using BS.
8	Li (2022)	Application of Machine Learning in Option Pricing: A Review	Machine learning, option pricing, deep learning, neural network.	European Options, American Options, Other Options	Advances in Economics, Business and Management Research	- This article reviews the use of various methods for pricing different options in recent years and compares their advantages and disadvantages.
9	Liu and Huang (2019)	Pricing Options and Computing Implied Volatilities Using Neural Networks	Machine learning; neural networks, computational finance, option pricing, implied volatility, GPU, Black-Scholes, Heston COS method, Brent's iterative root-finding method	-	Journal of Risks	The numerical results show that the ANN solver can significantly reduce the computing time.

3. Research methodology

3.1 Models and formulas

Different models and methods have been used to simulate option prices. To summarize and condense the content, we have omitted these formulas. Instead, we refer to each model as an article. These models include the Monte Carlo simulation (M-C), Black-Scholes option pricing model (BSM), binomial option pricing model (BOPM), Merton's Jump Diffusion option pricing model, Heston option pricing model, NIG-CIR option pricing model, GARCH option pricing model, genetic algorithm (GA), and BSANN option pricing model.

To evaluate option prices using a Monte Carlo simulation, this method generates different possible paths for the underlying asset in a risk-neutral environment. Then, the value of the option is calculated for each path. Finally, it takes the average of all the option values and calculates their present values. The Monte Carlo method employs three steps to price an option (Trinh and Hanzon, 2022).

1. Calculate the potential futures prices of underlying asset(s).
2. Calculates the payoff of the option for each potential underlying price path
3. Discount the payoff back to today and average them to determine the expected price

This process begins by computing the B-S numbers using input parameters, such as the strike price and interest rate. Subsequently, a random variable called epsilon was used as an input number. Finally, the option value and payoff were computed. The Black-Scholes model is primarily used to calculate the theoretical value of European-style options. It cannot be applied to American-style options because it can be performed before the maturity date.

The binomial option pricing model uses an iterative procedure that allows for the specification of nodes or points in time between the valuation date and the option's expiration date (Yeh & Lien, 2020). The binomial model is best represented using binomial trees, which are diagrams that illustrate the payoff and value of the option at different nodes throughout its lifespan. BOPM has several assumptions (He 2019). The binomial pricing model traces the evolution of the option price in discrete time under a risk-neutral measure. This measure ensures that the discounted price process is martingale.

The jump-diffusion model, introduced in 1976 by Robert Merton, is a model for stock price behavior that incorporates small day-to-day "diffusive" movements together with larger, randomly occurring "jumps" (Chowdhury & Jeyasreedharan, 2019). The inclusion of jumps allows for more realistic "crash" scenarios, rendering the standard dynamic replication hedging approach of the Black-Scholes model ineffective. This causes option prices to increase compared with the Black-Scholes model, which depends on investors’ risk aversion. This equation is based on the B-S formula.

The stock price is full of volatility and the interest rate is not fixed. Therefore, it would be preferable to use a model that considers this condition. One model that rejects constant volatility and uses a stochastic process is the Heston stochastic volatility model (Heston, 1993). This model assumes that the volatility is arbitrary. It is an extension of the Black-Scholes stochastic differential equation (SDE), where the volatility is derived from the Cox-Ingersoll-Ross (CIR) process (Cui et al., 2017). This model assumes that the distribution of asset returns is nonlognormal.

The NIG-CIR option pricing model was developed by Carr et al. in 2003. This is a combination of these two words. NIG stands for Normal Inverse Gaussian and CIR stands for the Cox-Ingersoll-Ross model (Kovachev, 2014). The model is defined under the risk-neutral measure, which was first introduced by Duan (1995) with a locally risk-neutral valuation relationship (LRNVR), in which the conditional variances and model parameters remain the same under both physical and risk-neutral measures. Duan's LRNVR has been widely used by finance researchers and practitioners to price options under the GARCH framework. However, as pointed out by Barone-Adesi (Jayaraman, 2022), empirical evidence shows that the restriction in Duan's LRNVR resulted in poor pricing and hedging performance.

Since the groundbreaking research by Bollerslev (1986) and Engle (1982), the GARCH volatility model family has been extensively utilized in empirical asset pricing and financial risk management. This is primarily because the likelihood function of asset returns in GARCH models can often be expressed in a closed form using observed data. Model parameters can be estimated using the maximum likelihood estimation (MLE) method (Smith, 2019), which is often a challenging task for most stochastic volatility models. Motivated by the success of GARCH models in fitting asset returns, Duan (1995) pioneered the application of GARCH models to SPX option pricing by proposing LRNVR. Duan et al. 's LRNVR has been reported in many studies.

One of the main applicable analytical software programs is MATLAB. Many script codes were prepared in the file. Thus, by accessing these m.files, it is possible to tune, that is, change and improve them based on your problem. In this study, MATLAB R2017a was used as the primary analytical tool. Another important software is the Option Pricing Excel add-in software. By adding this feature, input variables can be inserted, and it is possible to see the final output for different option groups, such as Asian and European groups. However, other software is used, such as the MG Soft Exotic Options Calculator, FinOptionsXL, and option-pricing packages.

3.1.1 Genetic algorithm

The genetic algorithm is a global search technique that utilizes various operations, including coding, decoding, mutation, and crossover, to discover an approximate optimal solution (Gen & Lin, 2023). Mutation and crossover are operators that create new and optimal solutions, respectively. Binary coding was used, and 17 bits of chromosomes were utilized. Of these, 12 bits are used to represent the existence ("1") or non-existence ("0") of the input variables, whereas the remaining five bits (2^5=32) are used to determine the number of neurons in the hidden layers. The GA parameters are presented in Table 4.

Table 4. GA parameters

Output Error	**Output Activation Function**	**Input Activation Function**	**Mutation Rate**	**Crossover Rate**	**Number of Generation**	**Population size**	**Max Iteration**
MSE	Logistic	Logistic	0.1	0.9	50	20	1000
Selection parents			**Mutation**		**Crossover**		
Roulette wheel method			Binary Method		One-point method		

We used 70% of the data for training, and the remaining portion was used for the validation and testing datasets. Initially, we used 100 epochs and then increased it to 1000. The GA process is as follows.

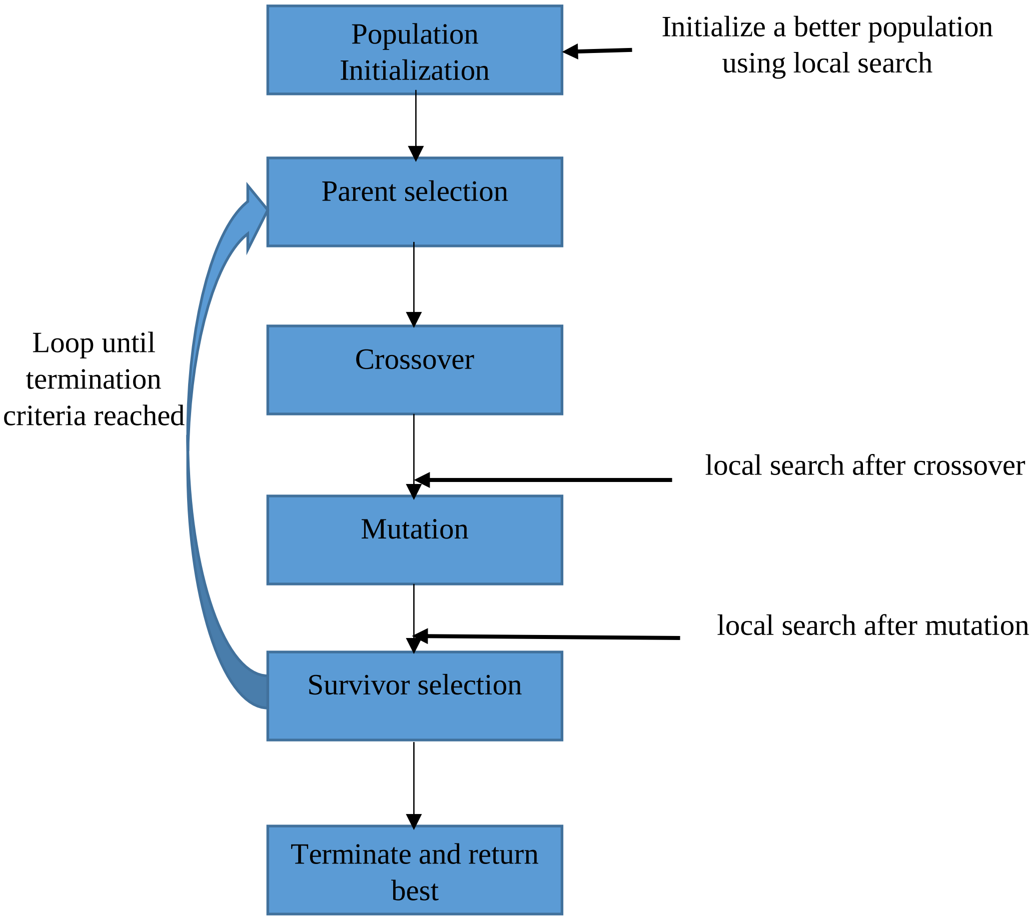


Figure 1. Genetic Algorithm Process

In this study, certain stock market indicators (specifically, peer stocks in the automobile industry) were utilized as input variables to enhance the predictive capability. We must normalize the data before using AI techniques. As mentioned earlier, the GA has been used for feature selection. However, it is a heuristic algorithm for optimizing and obtaining the best possible solution.

In this study, we attempt to predict the price of Khodro options as the dependent variable. Daily data were used. The considered indicators are listed in Table 5.

Table 5. Economic indicators as input variables

Row	Indicator	Symbols	Nature
1	Niroo moharekeh	Kemoharekeh	Input variable
2	Bahman group	Khebahman	Input variable
3	Bahman dizel	Khedizel	Input variable
4	Investment development of Irankhodro	Khegostar	Input variable
5	Saipa Dizel	Khekaveh	Input variable
6	Financial group of Kerman khodro	Khekerman	Input variable
7	Iranian tractor manufacturers	Khemotor	Input variable
8	Parskhodro	Khepars	Input variable
9	Saipa	Khesapa	Input variable
10	Iran automobile segment	Khetogha	Input variable
11	Zamyad	Khezamya	Input variable
12	IranKhodro	Khodro	Target variable

These indicators were adopted from the TSETMC database using the TSECLIENT software.

3.1.2 BSANN option pricing model

This section introduces and explains the BSANN model for option pricing. As you may know, artificial intelligence models attempt to emulate the intelligent processes of the human brain when making decisions. Based on different approaches or mechanisms, AI has various sub-branches such as machine learning, artificial neural networks, and deep learning (DL). In this study, artificial neural networks (ANN) were used to predict option prices. An ANN is a method that simulates the human brain and thinking approach. It has three important layers: I. input layer, II. Hidden layer, III. Output layer. In this study, B-S parameters such as stock price (S_0/K), time to maturity(τ), risk-free rate (r), and volatility (σ) were used as input variables for the artificial neural network (ANN). There are various types of information streams in networks. One of these is the feed-forward neural network. The information flow was direct and forward, without any feedback. The second is back propagation. In this data stream and network, the flow of information is accompanied by feedback and return. These two structures can be described as follows:

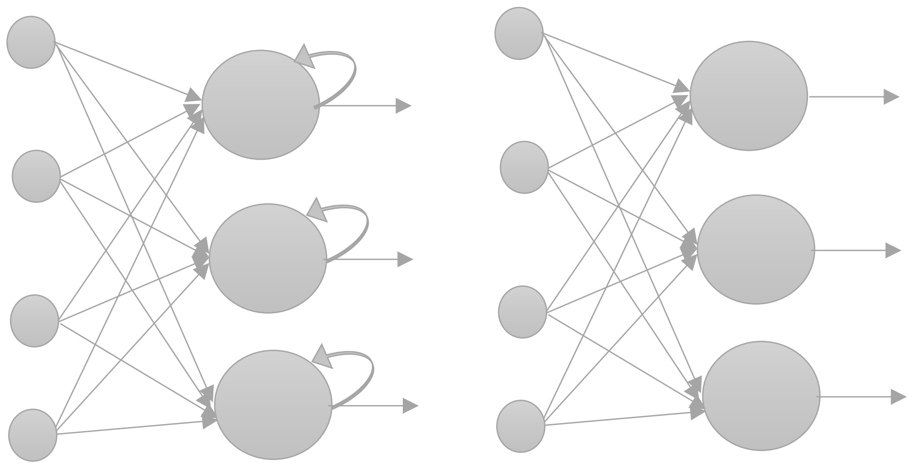


Figure 2. Backpropagation vs. feedforward neural network

A feed-forward multilayer perceptron neural network was used as the network structure. Machine learning includes three main components: training II. Validation III. Testing. Based on multiple studies, 70% of the data were used for training, while the remaining portion was allocated for validation and testing. In pricing options using ANN, the input and output variables need to be accessed. The underlying asset price (Khodro price) is an input variable, and the option price using the BS model is the output.

The following figure shows the network structure of the pricing option.

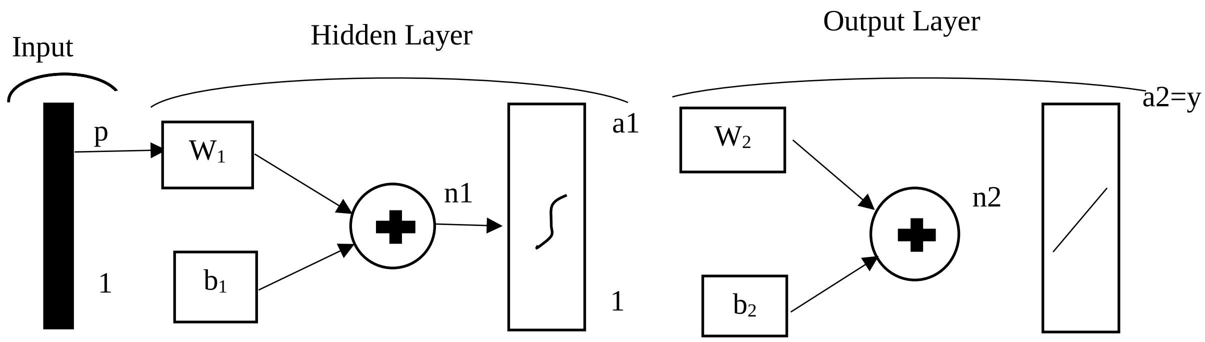


Figure 3. Architecture of the proposed Neural Network

Based on Figure 3, the first layer is the input layer. In this layer, there are weights and biases that are summed and then passed through the first activation function, which is used to capture nonlinear features. This process is then iterated in the next layer, which is the hidden layer. However, in this case, the new weights and biases must pass through a linear activation function. Finally, the output layer exists. Before performing any calculations, the data were normalized for several reasons. The most important reason for this is data scaling.

$$\widetilde{S_i} = \frac{(S_i - S_{min})}{S_{max} - S_{min}} . i = 1 .... N \quad (1)$$

where:

$\widetilde{S_i}$: Normalized data

S_i: Each observation of each variable

S_{min}: Minimum value of each variable

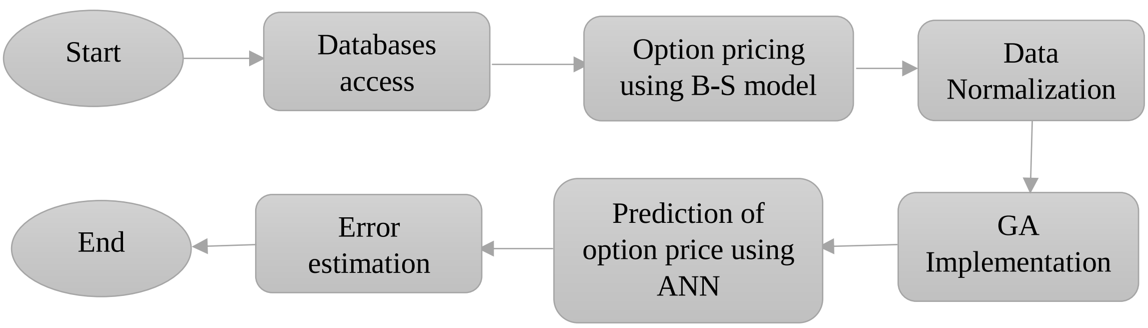
S_{max}: Maximum value of each variable

In Equation 1, i denotes the amount of data.
The different parameters and more details regarding the ANN parameters are available in the following table:

Table 6. Parameters

Parameters	Explanations
Training	Back-propagation (BP)
Optimization algorithm	Levenberg-Marquardt (LM)
Training rate	0.01
Iterations	1000
Activation function	Tan-Sigmoid
	Pure line

The Levenberg-Marquardt (LM) method is used as an optimization algorithm to minimize the error. First, the learning rate is set to 0.01. If we do not observe any improvement in the learning of the network, it increases to 0.1. The diagram below illustrates the research process and flowchart.



To calculate the option price, we need to consider the following parameters using the Black-Scholes (B-S) models.

Table 7. B-S parameters

Components	**Parameters**	**Range**	**Unit**
Input	Stock price (S_0/K)	[100, 600]	-
	Time to maturity (τ)	[0.002, 2]	Year
	Risk free rate (r)	[0.1, 0.2]	-
	Volatility (σ)	[0.4, 0.5]	-
Output	Call/ Put price	(0.0, 1)	-

Historical data shows that stock prices range from 100 to 600. Time to maturity was considered to be approximately two years. The minimum and maximum risk-free rates are 0.1 and 0.2, respectively. The volatility based on inflation ranges from 0.4 0.5. Finally, the option price is between 0 and 0.9, based on normalized data.

For more information about calculations and formulas using the Black-Scholes model, please refer to section 3-2-2.

4. Result and discussion

First, we determine parameters such as the price of the underlying asset (current stock price), strike price, interest rate, volatility, and time to expire. Table 8 lists the parameters.

Table 8. Parameters

Row	Symbol	Khodro
1	Current Stock Price (S_0)	400
2	Strike Price (K)	200:400:600
3	Interest Rate (Risk Free Rate)	0.2
4	Volatility (σ)	0.42
5	Expiry (Year)	1

We are going to price an option for a stock in the automobile industry with the symbol "Khodro" for a one-year period, which means the time to maturity is one year. The stock price is 400 Toman (or 4000 Rials), and the hypothetical strike prices for the call option and put option are 200 and 600 Toman, respectively. According to central bank reports, the interest rate, which represents the risk-free rate, is 20%. The stock market volatility was 42%.

We consider a strike price that is 50% different from the current price of the underlying asset, for various reasons.

1. The inflation rate was high. At the time of writing the paper, the inflation rate was 42% according to reports from the statistical center.
2. Devaluation of the national currency
3. There is high risk associated with investing in stocks and options. Therefore, taking high risks increases the expected profit.
4. Increasing the value of parallel markets, such as cryptocurrencies, foreign currencies, automobiles, housing, etc.

4.1. Monte Carlo Option Pricing Simulation

First, we should establish the parameters in various dimensions, such as strike price and volatility.

Table 9. Input variables and results

Input Variables	Input Values	
Underlying price	400	
Strike price	Call:200, Put:600	
Time to maturity (days)	360	
Interest rate (%)	20	
Volatility (%)	42	
Number of steps	36	
Number of simulations	100.000	
Output	**Results**	
Option	**Call**	**Put**
Option value	237.726784	128.015638
Standard deviation	16.581779	16.352008
Standard error	0.537096	0.517096

Elapsed Time	32.2391	30.9944

Then, you can observe the simulation results after 100.000 iterations in the following figure:

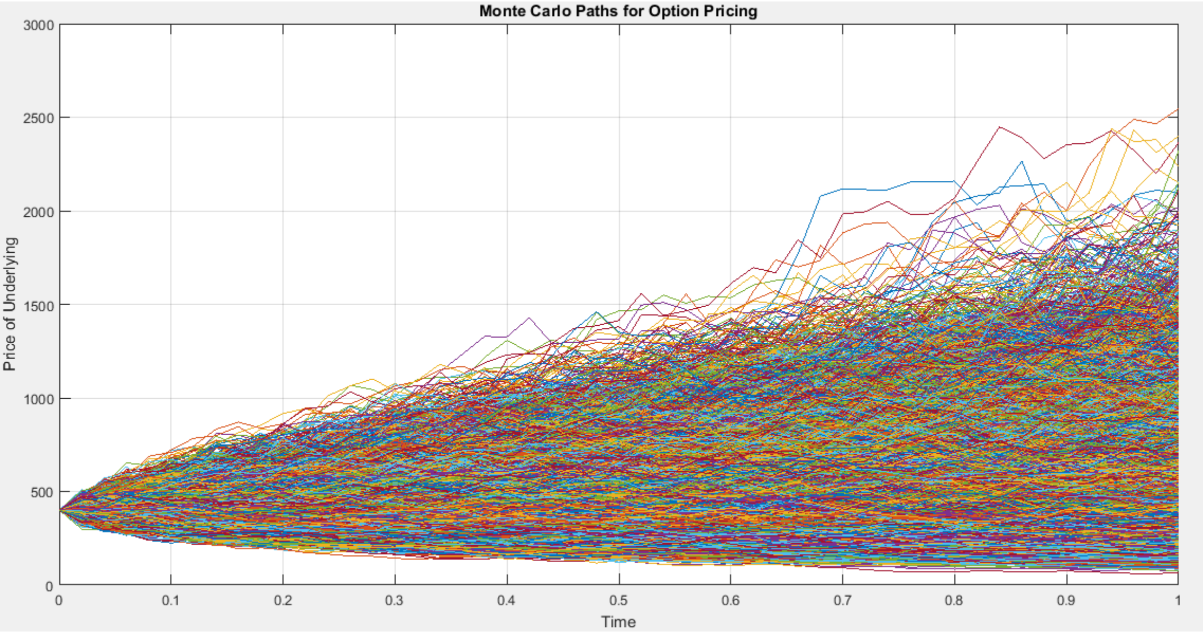


Figure 5. Monte Carlo simulation results after 100,000 iterations

We created an m-file in MATLAB with specified parameters including the input price, number of iterations, and asset paths. To accomplish this, we utilized a preexisting m-file or script. We then changed or adjusted the scale according to the desired outcomes. The underlying asset price for the initial days does not exhibit much volatility or increase significantly. During this time, the underlying assets increased to 2000 and 2500. The following table shows the value of stock options due to ROI during different months.

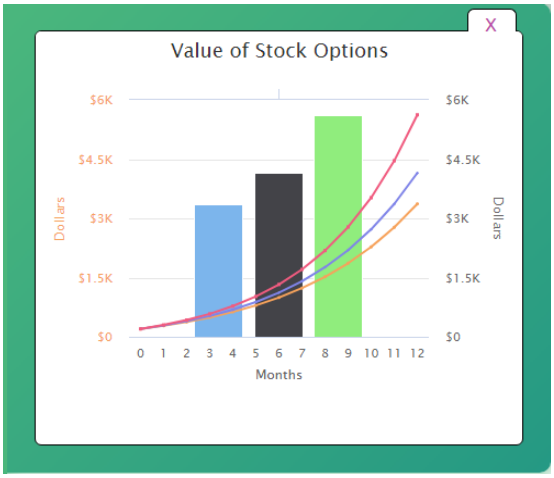


Figure 6. ROI of Stock Option Over Time (Call option)

Stock Option Values by Month

Month	At 20% Growth	At 22% Growth	At 25% Growth
1	$280.00	$288.00	$300.00
2	$376.00	$395.36	$425.00
3	$491.20	$526.34	$581.25
4	$629.44	$686.13	$776.56
5	$795.33	$881.08	$1,020.70
6	$994.39	$1,118.92	$1,325.88
7	$1,233.27	$1,409.08	$1,707.35
8	$1,519.93	$1,763.08	$2,184.19
9	$1,863.91	$2,194.96	$2,780.23
10	$2,276.69	$2,721.85	$3,525.29
11	$2,772.03	$3,364.66	$4,456.61
12	$3,366.44	$4,148.89	$5,620.77

Figure 7. Stock Option Value by Month (Call option)

Figure 8. ROI of Stock Option Over Time (Put option)

Stock Option Values by Month

Month	At 20% Growth	At 22% Growth	At 25% Growth
1	$0.00	$0.00	$0.00
2	$0.00	$0.00	$25.00
3	$91.20	$126.34	$181.25
4	$229.44	$286.13	$376.56
5	$395.33	$481.08	$620.70
6	$594.39	$718.92	$925.88
7	$833.27	$1,009.08	$1,307.35
8	$1,119.93	$1,363.08	$1,784.19
9	$1,463.91	$1,794.96	$2,380.23
10	$1,876.69	$2,321.85	$3,125.29
11	$2,372.03	$2,964.66	$4,056.61
12	$2,966.44	$3,748.89	$5,220.77

Figure 9. Stock Option Value by Month (Put option)

The value of the stock options was divided into three parts, with ROI of 20%, 22%, and 25%. After a year, the value of stock options increased by more than 5000 Rials for both call and put options.
For a call option, the estimated value of the options is 3,366.44, after 12 months, with a 20% growth rate. In a put option, the options are estimated to be worth 2,966.44 after 12 months, with a 20% growth rate.

Finally, the payoff is calculated for four positions: long, short, long, and short. The following figures show this:

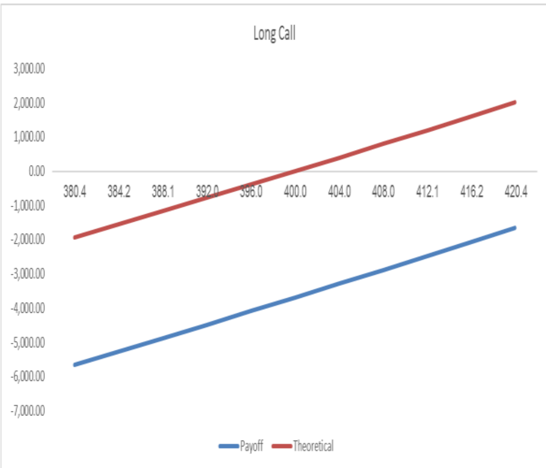


Figure 10. Long Call

Figure 11. Short Call

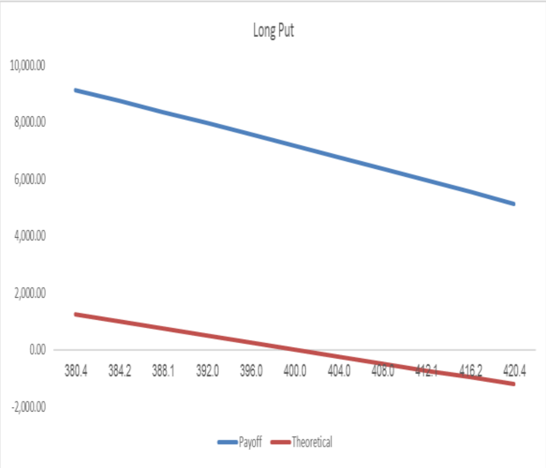


Figure 12. Long Put

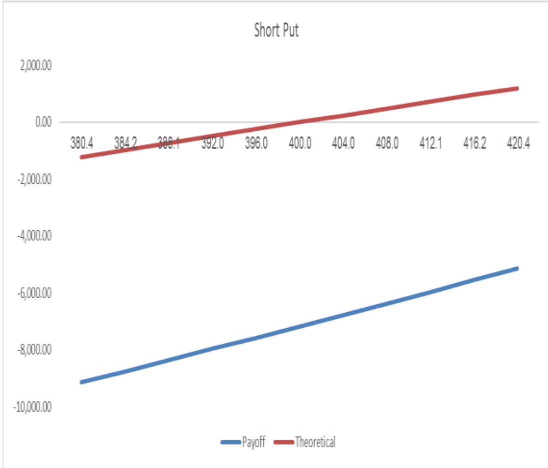


Figure 13. Short Put

More details on the payoff, theoretical value, and stock price over time can be observed in the following table.

Table 10**.** Some details about long call, short call, long put and short put positions

Payoff	-5,649.25	-5,265.01	-4,876.89	-4,484.85	-4,088.85	-3,688.85	-3,288.85	-2,884.85	-2,476.81	-2,064.69	-1,648.45
Theoretical	-1,938.0	-1,558.7	-1,175.3	-787.7	-396.0	0.0	396.2	796.7	1,201.3	1,610.2	2,023.4
Stock Price	380.4	384.2	388.1	392.0	396.0	400.0	404.0	408.0	412.1	416.2	420.4
Payoff	5,649.25	5,265.01	4,876.89	4,484.85	4,088.85	3,688.85	3,288.85	2,884.85	2,476.81	2,064.69	1,648.45
Theoretical	1,938.0	1,558.7	1,175.3	787.7	396.0	0.0	-396.2	-796.7	-1,201.3	-1,610.2	-2,023.4
Stock Price	380.4	384.2	388.1	392.0	396.0	400.0	404.0	408.0	412.1	416.2	420.4
Payoff	9,133.19	8,748.95	8,360.83	7,968.79	7,572.79	7,172.79	6,772.79	6,368.79	5,960.75	5,548.63	5,132.39
Theoretical	1,239.8	989.9	740.8	492.8	245.8	0.0	-242.2	-483.0	-722.6	-960.7	-1,197.3
Stock Price	380.4	384.2	388.1	392.0	396.0	400.0	404.0	408.0	412.1	416.2	420.4
Payoff	-9,133.19	-8,748.95	-8,360.83	-7,968.79	-7,572.79	-7,172.79	-6,772.79	-6,368.79	-5,960.75	-5,548.63	-5,132.39
Theoretical	-1,239.8	-989.9	-740.8	-492.8	-245.8	0.0	242.2	483.0	722.6	960.7	1,197.3
Stock Price	380.4	384.2	388.1	392.0	396.0	400.0	404.0	408.0	412.1	416.2	420.4

4.2 Black-Scholes Option Pricing Model (BSM)

First, the parameters are set. Thus, it was possible to obtain results. Table11 shows the parameters and results.

Table 11. Input variables and results for call and put option using the BSM model

Input Variables	Input Values
Underlying price	400
Strike price	Call:200, Put:600
Time to maturity (days)	360
Interest rate (%)	20
Volatility (%)	42
Dividend Yield	0.01

Results

Snapshot	**Calls**	**Puts**	**Snapshot**	**Calls**	**Puts**
Price	236.889	0.635	**Price**	37.034	128.272

Delta	0.990	-0.010	**Delta**	0.390	-0.610
Gamma	0.000	0.000	**Gamma**	0.002	0.002
Theta	-0.093	-0.004	**Theta**	-0.153	0.116
Vega	0.110	0.110	**Vega**	1.538	1.538
Rho	1.585	-0.044	**Rho**	1.202	-3.686
Position	ITM	OTM	**Position**	OTM	ITM
Elasticity	1.67	-6.13	**Elasticity**	4.21	-1.90
Probability of closing ITM	97.2%	2.8%	**Probability of closing ITM**	24.2%	75.8%

It should be noted that all parameters and their values were adapted from various research papers and statistics. For example, volatility and interest rates are determined by the inflation rate, which is set by the central bank. In all tables, the appropriate parameters must be set, and the results can be seen automatically. However, interpreting these results is important. From the formulas, it is clear that gamma has the same value for both calls and puts, and the same applies to vega for call and put options. This can be seen directly from put-call parity because the difference between a put and a call is a forward contract, which is linear in and independent of (a forward contract has zero gamma and zero vega).

In practice, some sensitivities are often expressed in scaled-down terms to align with the potential magnitude of the parameter variations. For example, it is often reported to be divided by 10,000 (1 basis point rate change), 100 (1 vol point change), and 365 or 252 (1-day decay based on either calendar days or trading days per year). A 1-year call option with an exercise price (strike price) of 200 on stock trading at 400. Determine whether you should buy the option if the annual risk-free rate is 20% and the annual standard deviation of stock returns is 42%.

The value of the call option needs to be determined using the Black-Scholes option pricing model and then compared with the current price of the option. If an option is fairly priced, it can be purchased.

We first need to find d_1 and d_2:

$$d_1 = \frac{\ln(400/200)+(20\%+0.5*42\%^2)(1)}{42\%\sqrt{(1)}} = 2.336$$

$$d_2 = \frac{\ln(400/200)+(20\%-0.5*42\%^2)(1)}{42\%\sqrt{(1)}} = 2.336 - 42\%\sqrt{1} = 1.916$$

Next, we determined the probability of a standardized normal distribution using the NORMSDIST function in Microsoft Excel. $N(d1)$ and $N(d2)$ 0.990, and 0.972, respectively.

Once we have $N(d1)$ and $N(d2)$, we can plug-in the relevant numbers in the Black-Scholes formula (Table5):

$$C = 400 * 0.990 - 200 * e^{-0.2*1} 0.972$$

$$C = 236.889$$

As per the model, the option value is higher than the strike price. Therefore, the call option is invalid. If we have the current value of the call premium, stock price, exercise price, time to maturity, and risk-free rate, we can calculate implied volatility. After finding the call and put options, we calculate the payoff. Figure 14 and 15 present the payoffs and behavior of the option during the considered period.

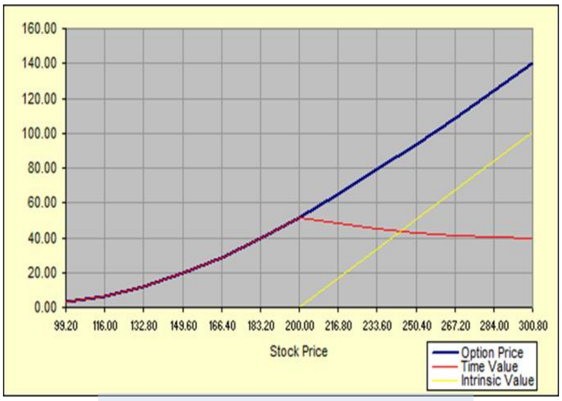


Figure 14. Call Option price

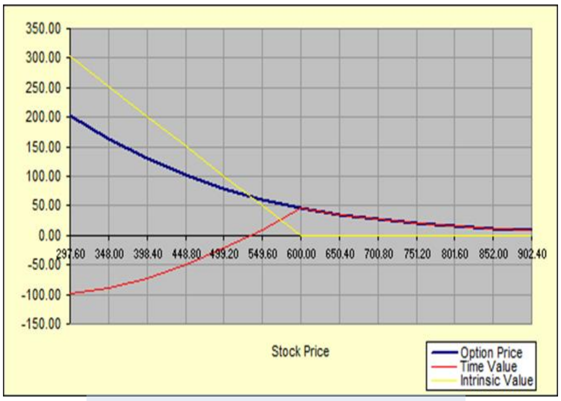


Figure 15. Put Option price

Figure 14 shows the value of the call option with an underlying price of 400 Toman and strike price of 200. If the value of the option is greater than 200, the option is valid. If the price is below 200, the value is zero, and no option is applied. Figure 15 shows the value of the put option with an underlying price of 400 and strike price of 600. If the value of the option is less than 600, then the option is valid. If it is greater than 600, the value is zero, and no option is applied.

4.3 Binomial Option Pricing (BOP)

Several steps were taken to calculate the option price using the BOP model. To simplify matters, we make several assumptions:

1 .Input in blue cells. The output appears in yellow cells.
2 .The macro uses a binomial tree to price standard options.
3 .The assumptions are the GBM and risk-neutral valuation.
4 .No tax or transaction costs were included.
5 .The maximum number of steps is 255 .
6. For reference, please refer to Hull J. (2000).

The parameters considered for calculating the call option are as follows:

Table 12. Input variables and results for pricing European call option using the BOPM model

Input variables			Results	
European or American	E or A	E	**Option Value**	232,963927
Call or put	C or P	C	**Delta**	0,98019234
Current Asset Price	S (0)	400	**Gamma**	0,0001667
Strike Price	X	200	**Pheta**	-30,52298
Time to Maturity	T	1	(Greek letters need at least 3 steps.)	
Volatility	Sigma	0,42		
Risk-Free Interest Rate	R	0,2		
Dividend Yield	Q	0,01		
No of Steps	N	20		

Here, is an example of the binomial options pricing model for a 20 period (N) call option. Let's say the current stock price (S) is 400. The strike price (X) of the option is 200. The option expires (time to maturity) in one year. At the end of the year, the stock price will either rise to 2616 or fall to 61. We assume that there is a 98% chance that it will rise to 2616 and a 2% chance that it will fall to 61. The interest rate (R) is 20%. If the stock rises to 2616, the value of the option will be 2417. This is

because the option value is equal to the stock price minus strike price. There is a 98% chance that the option will be worth 2417 at the end of the year. If the stock falls to 61, then the value of the option will be 0. This is because option value cannot be negative. The values were either positive or zero. There is a 2% chance that the option will be worth nothing by the end of the year.
We can then calculate the call option using the BOPM.

Table 13. An example of calculating the price of a European call option using the Binomial Option Pricing Model (BOPM) with 20 steps.

Binomial Option Pricing Model (BOPM)-Call option										
Time step	**0(Today)**	**1**	**2**	**3**	**4**	**5**	**…**	**18**	**19**	**20**
Asset price	400	439/3864852	482/6512	530/176	582/3805	639/7253	…	2168/806	2382/36	2616/942
		364/1441086	400	439/3865	482/6512	530/176	…	1797/41	1974/395	2168/806
			331/5023	364/1441	400	439/3865	…	1489/614	1636/291	1797/41
				301/7866	331/5023	364/1441	…	1234/527	1356/086	1489/614
					274/7345	301/7866	…	1023/121	1123/864	1234/527
						250/1074	…	847/9175	931/4088	1023/121
							…	702/7166	771/9104	847/9175
								582/3805	639/7253	702/7166
								482/6512	530/176	582/3805
								400	439/3865	482/6512
								331/5023	364/1441	400
								274/7345	301/7866	331/5023
								227/6878	250/1074	274/7345
								188/6976	207/2779	227/6878
								156/3842	171/7828	188/6976
								129/6043	142/366	156/3842
								107/4104	117/9866	129/6043
								89/01696	97/78212	107/4104
								73/77332	81/0375	89/01696
									67/1603	73/77332
										61/14007
Option value	232/963927	270/1691051	311/4101	357/004	407/3247	462/8063	…	1970/598	2183/159	2416/942
		196/4171036	229/9116	267/1741	308/4648	354/0988	…	1599/574	1775/398	1968/806
			163/2322	193/2361	226/7973	264/1215	…	1292/086	1437/463	1597/41
				133/2355	159/9091	189/9867	…	1037/253	1157/398	1289/614
					106/3156	129/7594	…	826/0587	925/2922	1034/527
						82/4261	…	651/0303	732/9332	823/1211
							…	505/9745	573/5146	647/9175
							…	385/7586	441/3955	502/7166
								286/1291	331/9011	382/3805
								203/5605	241/1569	282/6512
								135/1313	165/9521	200
								78/42015	103/6257	131/5023
								33/89614	51/97237	74/73449
								7/545527	14/45403	27/68781
								0	0	0
								0	0	0
								0	0	0
								0	0	0
								0	0	0
									0	0
										0

The option price varied for each step. The value of the option at each step can be seen in the following steps:

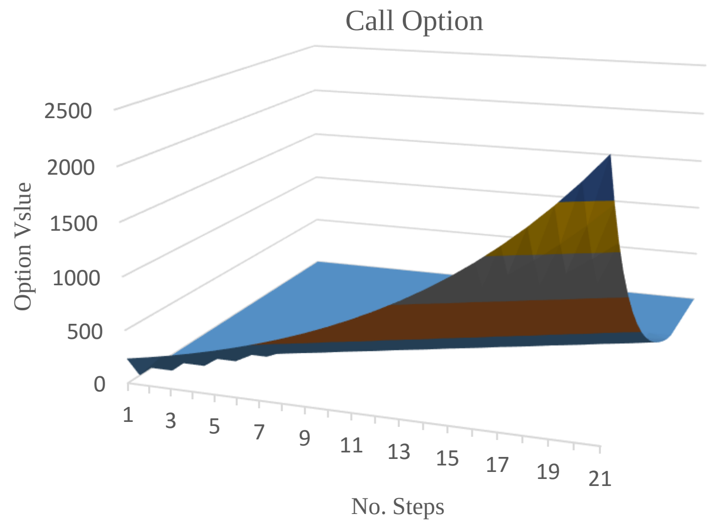


Figure 16. Call Option price, using the BOPM

We use the same steps to calculate the put option. The considered parameters are listed in Table 14.

Table 14. Input variables and results for pricing European put options using the BOPM model

Input variables			Results	
European or American	E or A	E	**Option Value**	130,070684
Call or put	C or P	P	**delta**	-0,6229038
Current Asset Price	S (0)	400	**gamma**	0,0022917
Strike Price	X	600	**pheta**	41,9184538
Time to Maturity	T	1	(Greek letters need at least 3 steps.)	
Volatility	Sigma	0,42		
Risk-Free Interest Rate	R	0,2		
Dividend Yield	Q	0,01		
No of Steps	N	20		

Here is an example of the binomial option pricing model for a 20 period (N) put option. Let's say the current stock price (S) is 400. The strike price (X) of the option is 600. The option expires (time to maturity) in one year. At the end of the year, the stock price will either rise to 2616 or fall to 61. We assume that there is a 38% chance that it will rise to 2616 and a 62% chance that it will fall to 61. The interest rate (R) is 20%.

If the stock rises to 2616, then the value of the option will be 0. This is because option value cannot be negative. The values were either positive or zero. There is a 38% chance that the option will be worth nothing by the end of the year. If the stock falls to 61, then the value of the option will be 539. This is because the option value equals the strike price minus stock price. There is a 62% chance that the option will be worth 539 by the end of the year.

The next step is to calculate the put option using a Binomial Option Pricing Model (BOPM). Table 15. An example of calculating the price of a European put option using the Binomial Option Pricing Model (BOPM) with 20 steps

Binomial Option Pricing Model (BOPM)-Put option										
Time step	**0(Today)**	**1**	**2**	**3**	**4**	**5**	**…**	**18**	**19**	**20**
Asset price	**400**	439/3865	482/6512	530/176	582/3805	639/7253	…	2168/806	2382/36	2616/942
		364/1441	400	439/3865	482/6512	530/176	…	1797/41	1974/395	2168/806
			331/5023	364/1441	400	439/3865	…	1489/614	1636/291	1797/41
				301/7866	331/5023	364/1441	…	1234/527	1356/086	1489/614
					274/7345	301/7866	…	1023/121	1123/864	1234/527
						250/1074	…	847/9175	931/4088	1023/121
							…	702/7166	771/9104	847/9175
								582/3805	639/7253	702/7166
								482/6512	530/176	582/3805
								400	439/3865	482/6512
								331/5023	364/1441	400
								274/7345	301/7866	331/5023
								227/6878	250/1074	274/7345
								188/6976	207/2779	227/6878
								156/3842	171/7828	188/6976
								129/6043	142/366	156/3842
								107/4104	117/9866	129/6043
								89/01696	97/78212	107/4104
								73/77332	81/0375	89/01696
									67/1603	73/77332
										61/14007
Option value	**130/0706838**	109/2222	88/85502	69/57089	51/99032	36/68512	…	0	0	0
		156/091	134/2625	112/2542	90/65987	70/16753	…	0	0	0
			183/7577	161/6657	138/7277	115/4451	…	0	0	0
				212/3064	190/6886	167/6472	…	0	0	0
					240/9333	220/4437	…	0	0	0
						268/9104	…	0	0	0
							…	3/859326	0	0
								34/31332	8/246181	0
								105/9504	64/11888	17/61953
								188/519	154/8631	117/3488
								256/9482	230/0678	200
								313/6593	292/3942	268/4977
								360/659	344/0476	325/2655
								399/6102	386/8556	372/3122
								431/8913	422/333	411/3024
								458/6444	451/7351	443/6158
								480/8162	476/1022	470/3957
								499/1912	496/2967	492/5896
								514/4196	513/0329	510/983
									526/9032	526/2267
										538/8599

The put option price for each step is shown in Figure 17.

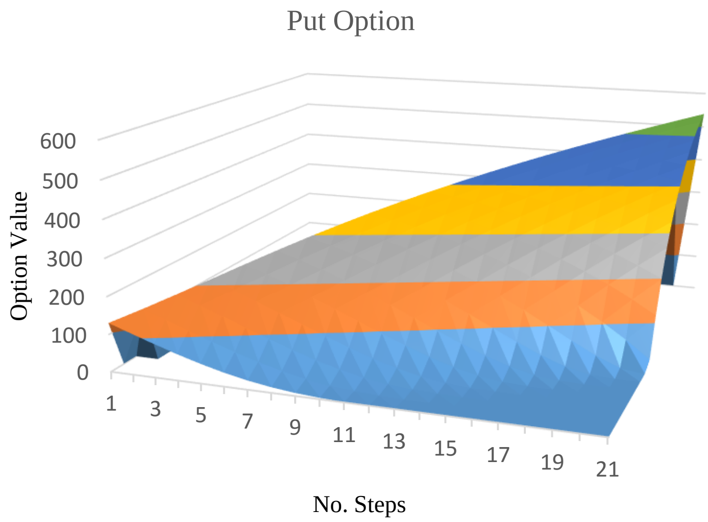


Figure 17. Put Option price, using the BOPM

We calculate the option price using the Binomial Options Pricing Model (BOPM) with a maximum of 255 iterations. The results are presented in the appendix (Table A1). A related macro is used for the calculation of BOPM in the appendix (Table A2).

4.4 Merton's Jump Diffusion Option Pricing Model

First, we begin this section by setting the parameters, as in other sections. The table below shows the primary and associated parameters.

Table 16. input variables and the results

Input Variables	Input Values	
Underlying price	400	
Strike price	Call:200, Put:600	
Time to maturity (days)	360	
Interest rate (%)	20	
Volatility (%)	42	
Iteration	30	
Jump Per Year	1	
Percent Total Vol	0.5	
Dividends Rate	0.01	
Output	**Results**	
Option	**Call**	**Put**
Option value	232.383	130.2655
Standard deviation	12.581779	13.352008
Standard error	0.41939	0.4450667
Elapsed Time	7.12	7.12

For the call and put options, the option price and standard error were 232.383 and 130.2655, with standard errors of 0.41939 and 0.445066, respectively.

There are more details on these jumps and price levels. Both the drift and jump values follow a normal distribution. The vertical lines in the plot indicate the jump positions. These parameters are listed in the following table along with their corresponding equations.

$$r_t = \alpha + \varepsilon_t + I_t u_t$$

Table 17. details about jumps

Variable	Value	Symbol
Constant Drift	0.10%	α
Std Dev of Drift	42.00%	σ_ε
Jump Probability	10.00%	-
Jump Mean	1.00%	E[u]
Jump Std Dev	35.00%	σ_u

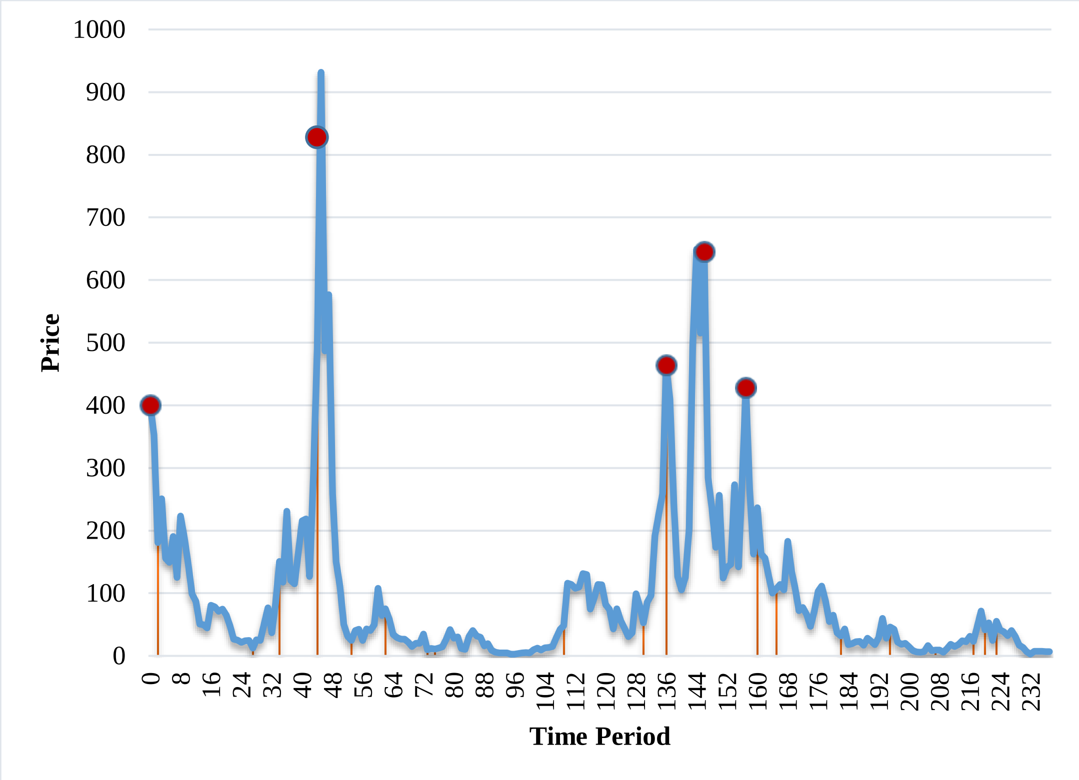


Figure 18. Jump diffusion

In the figure above, there are five main jumps: a jump at 400, two jumps between 400 and 500, a jump between 600 and 700, and a jump between 900 and 1000 price levels.

The following figures show the volatility surface of call options and put options in comparison to the Black-Scholes model using a spot price of 100 (on a 100 scale).

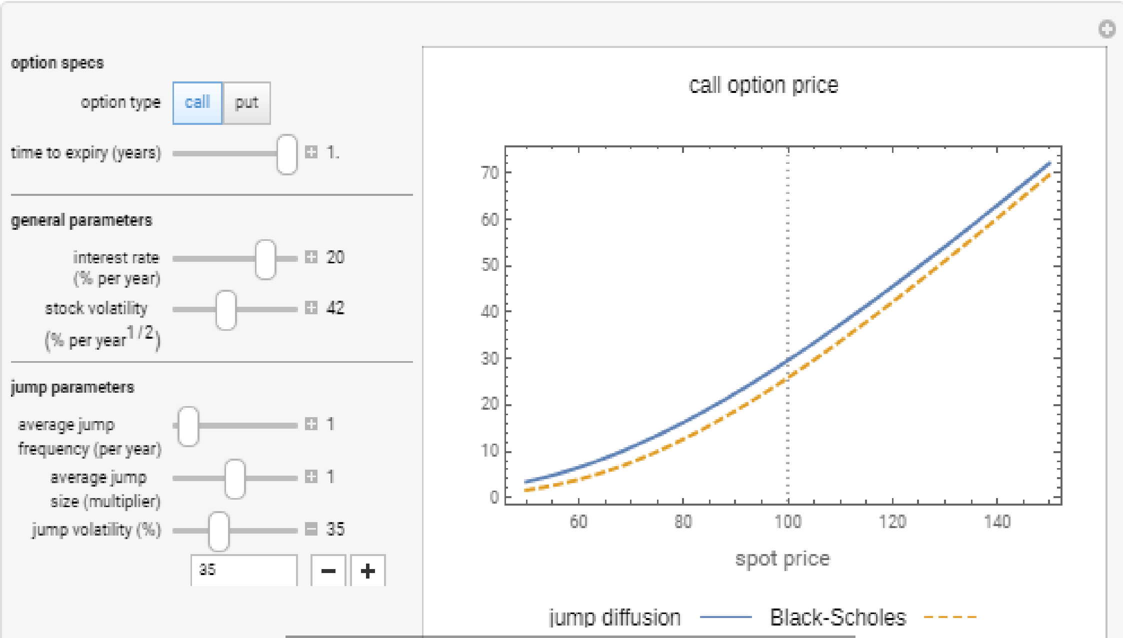


Figure 19. Call Option Price

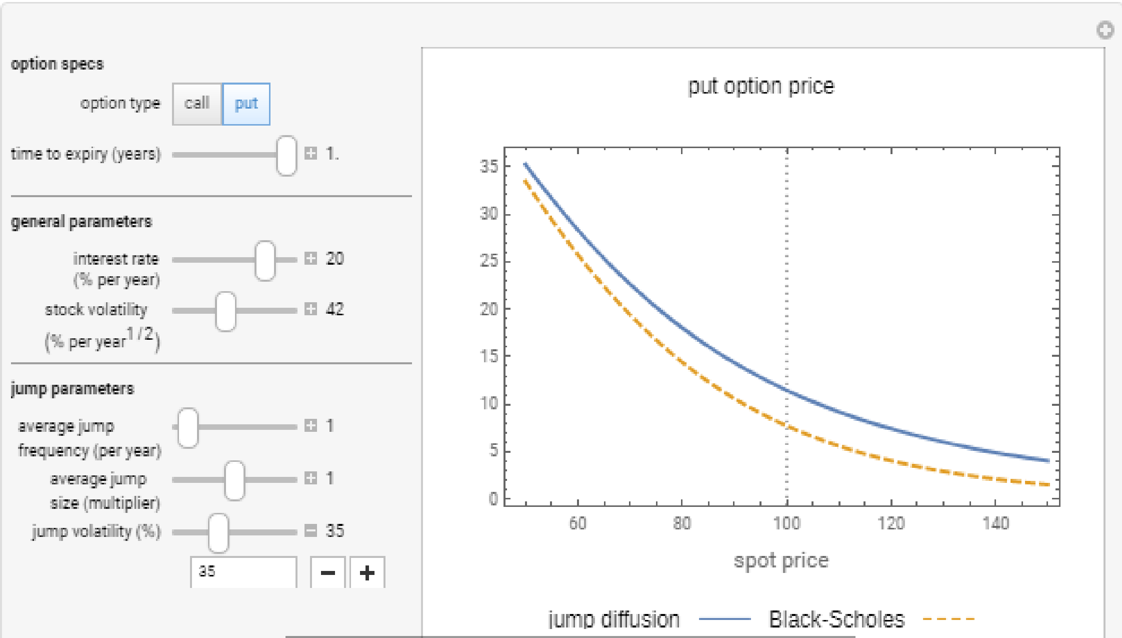


Figure 20. Put Option Price

For a more comprehensive and meaningful understanding of implied volatility, value probability density, and returns for future years, please refer to the appendix (Table A3) for additional results.

4.5 Heston Option Pricing Model

First, we considered the parameters and their corresponding values.

Table 18. Input variables and the results

Input Variables	Input Values
Underlying Price	400
Strike Price	Call=200, Put=600
Interest Rate (%)	20
Time to Maturity (Year)	1

Kappa (mean reversion speed of variance of base parameter set)	0.1	
Theta (volatility of variance of base parameter set)	0.42	
Rho (correlation of base parameter set)	0.1	
V_{inst} (instantaneous variance of base parameter set)	0.17	
V_{long} (long term variance of base parameter set)	0.17	
Dividends Rate	0.01	
Output	**Results**	
Option	**Call**	**Put**
Option Value	232.9843	133.5824
Standard Deviation	0.11245	0.11358
Standard Error	0.3748	0.3786
Elapsed Time	56.2391	53.9944

The option price and standard error for the call and put options were 232.98 and 133.58, with standard errors of 0.3748 and 0.3786, respectively.

The following figures show the volatility surfaces of call options and put options on a logarithmic scale:

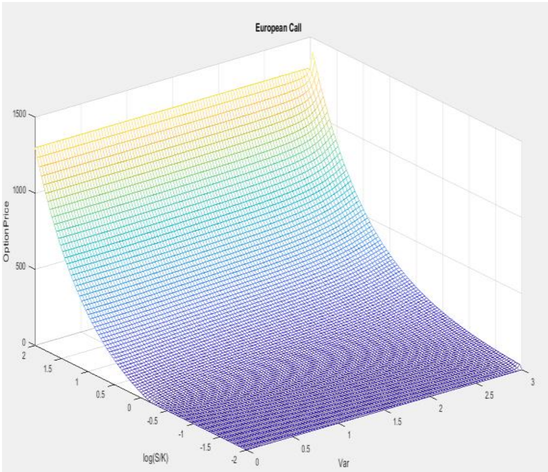


Figure 21. Call Option

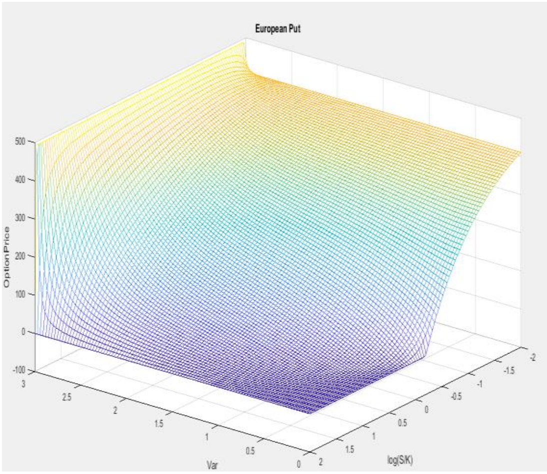


Figure 22. Put Option

4.6 NIG-CIR Option Pricing Model

The parameters of the model are as follows.

Table 19. Input variables and the results

Input Variables	Input Values
Underlying price	400
Strike price	Call:200, Put:600
Time to maturity (days)	360
Interest rate (%)	20
Volatility (%)	42
Dividends	0.01
Alpha	7
Beta	-3
Delta	1
Lambda	1

Kappa		1	
Output		**Results**	
Option	**Call**		**Put**
Option value	233.175		131.1045
Standard deviation	10.535879		10.624428
Standard error	0.351193		0.3541466
Elapsed Time	20.61		15.32

The option price and standard error for the call and put options are 233.175, and 131.1045 while the standard errors are 0.351193 and 0.3541466, respectively.

A figure of a normal inverse Gaussian process with certain parameters below shows skewness and kurtosis.

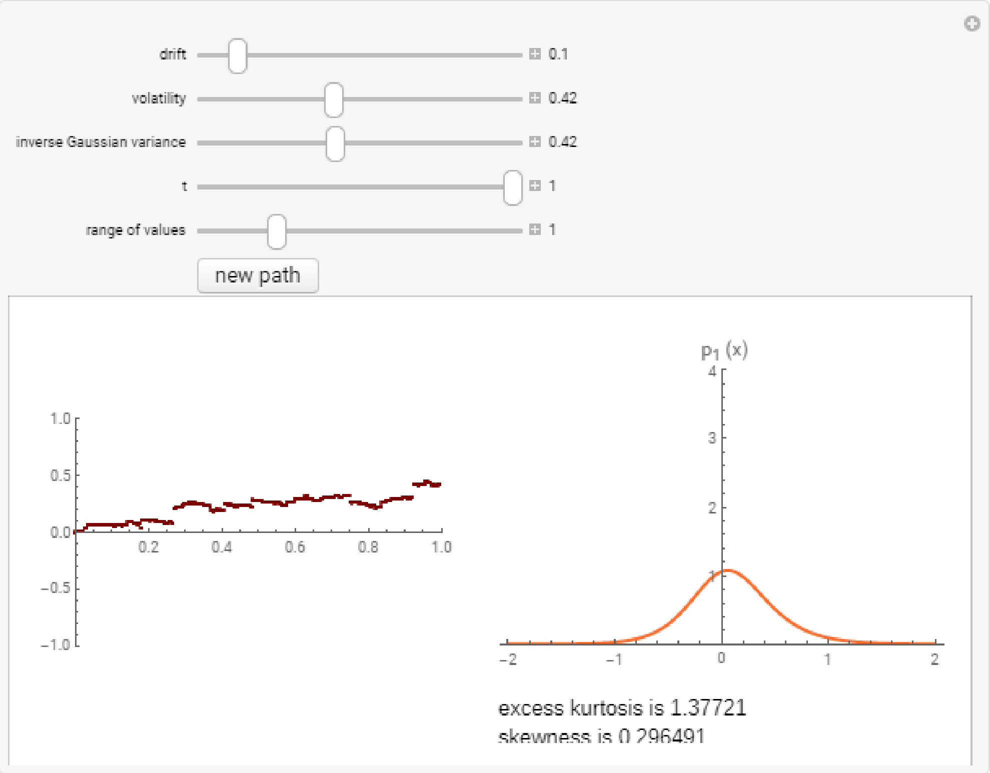


Figure 23. Normal inverse Gaussian process

The figure displays excess kurtosis and skewness.

4.7 GARCH estimation model

First, similar to the other methods, we begin the computation by setting the parameters.

Table 20. GARCH Model Parameters

Input Variables	Input Values
Underlying price	400
Strike price	Call:200, Put:600
Time to maturity (days)	365
Expected rate of return (%)	50
Volatility (%)	42
Alpha	7
Beta	-3
Omega	0.8
Number of time interval	1

Output	Results	
Option	**Call**	**Put**
Option value	286.2185	181.8312
Average value	188.9305046	275.799135
Standard deviation	108.4465426	84.76613162
Standard error	0.39895	0.62862
Elapsed Time	8.36	10.53

The initial price (underlying asset price) was 400. Therefore, we set the strike price to 200 for the call option, and 600 for the put option. The results for the call and put options are shown in Figure 24 and 25, respectively.

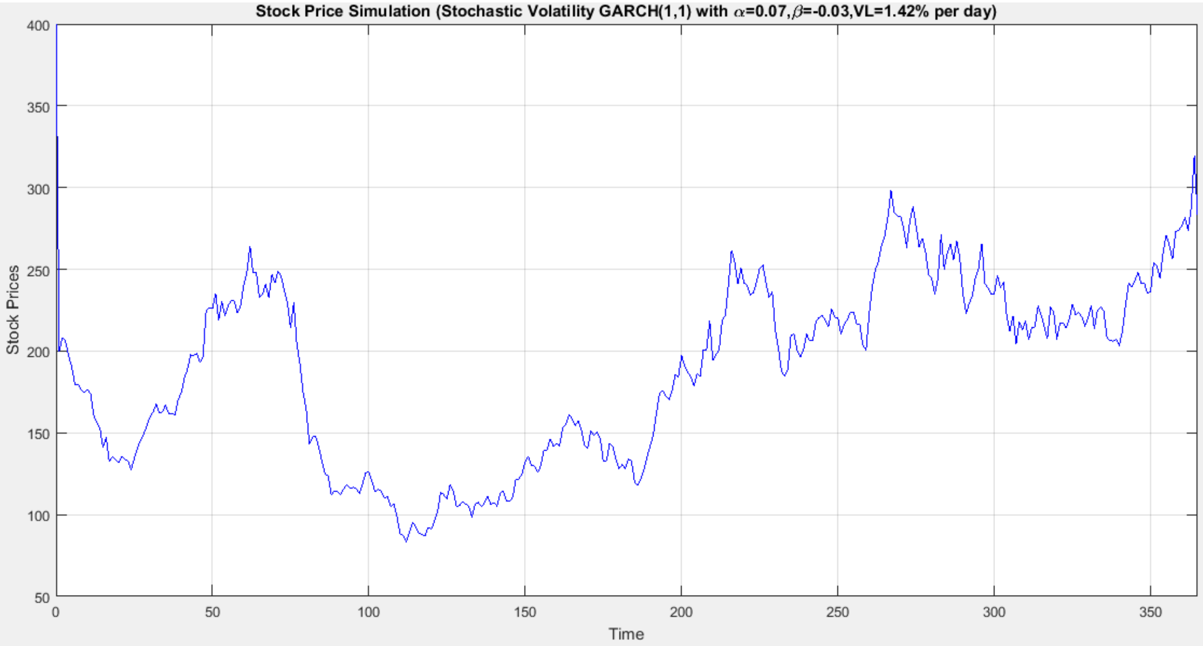


Figure 24. Call option calculation using GARCH model

Figure 25. Put option calculation using GARCH model

4.8 Genetic Algorithm Feature Selection

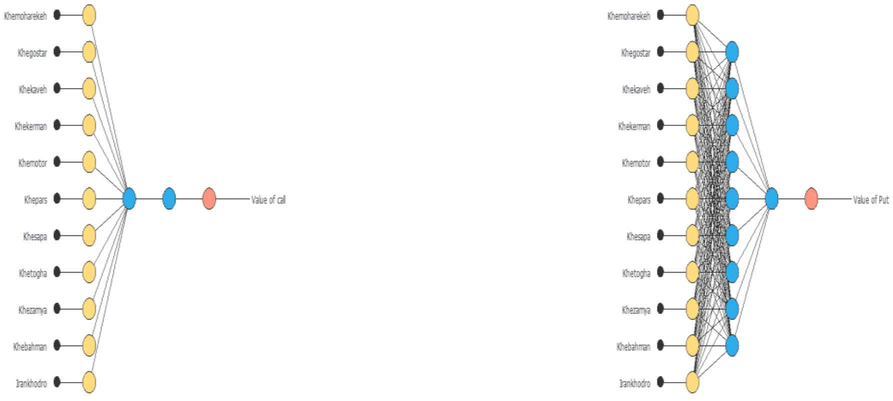
As mentioned earlier, GA was used for feature selection. Model selection was applied to find a neural network with a topology that optimizes the error of the new data. There are two different types of algorithms for model selection: order- and input-selection algorithms. Order selection algorithms are used to determine the optimal number of hidden neurons in the network. Input selection algorithms are responsible for determining the optimal subset of the input variables.

The algorithm selected for order selection in this application was SA. This method was inspired by the metallurgical industry and utilizes stochastic principles. A graphical representation of the resulting deep architecture is presented below. It contains a scaling layer, a neural network, and an unscaling layer. Yellow circles represent scaling neurons, blue circles represent perceptron neurons, and red circles represent unscaled neurons. The number of inputs was 11 and the number of outputs was one. The complexity, represented by the number of hidden neurons, is 1.

Table 21**.** Order selection algorithm parameters

	Description	Value
Minimum order	Number of minimum hidden perceptrons to be evaluated.	1
Maximum order	Number of maximum hidden perceptrons to be evaluated.	10
Cooling Rate	Temperature reduction factor for the simulated annealing.	0.5
Trials number	Number of trials for each neural network.	3
Tolerance	Tolerance for the selection error in the trainings of the algorithm.	0.01
Selection loss goal	Goal value for the selection error.	0
Minimum temperature	Minimum temperature reached in the simulated annealing algorithm.	0.001
Maximum iterations number	Maximum number of iterations to perform the algorithm.	100
Maximum time	Maximum time for the order selection algorithm.	3600
Plot training error history	Plot a graph with the training error of each iteration.	TRUE
Plot selection error history	Plot a graph with the selection error of each iteration.	TRUE

This table can be applied to two options: 1. Call option 2. Put option.
The following figure illustrates the final architecture for the order selection.



Order selection for call option

Order selection for put option

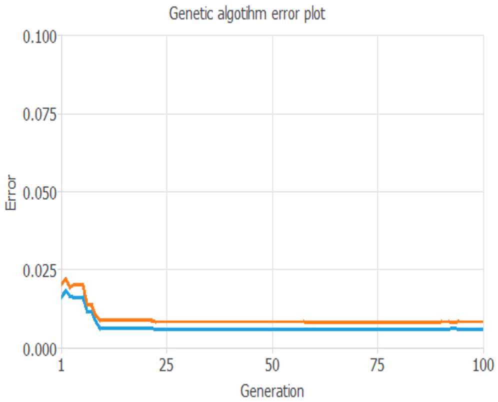
Figure 26. Order selection results

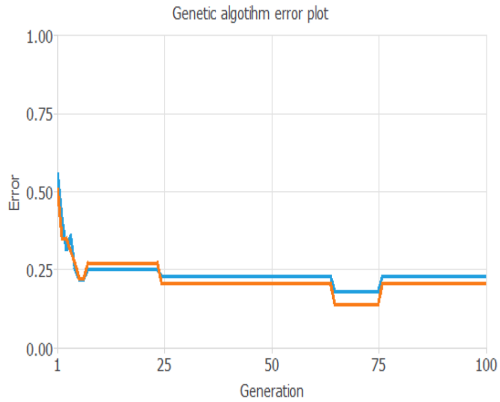
For call and put options, the best network structures were [11-1-1] and [11-9-1], respectively.

Table 22. order selection results

Criteria	Call	Put
Optimal order	1	9
Optimum training error	0.006001	0.117048
Optimum selection error	0.008186	0.102574
Iterations number	4	7
Elapsed time	0:01	0:00

The following chart displays the error history of various subsets during the GA input-selection process. The blue line indicates the training error. The initial values for the call and put options are 0.0162654 and 0.55321, respectively. The final value after 100 generations was 0.00602875 for the call option and 0.22185 for the put option. The orange line represents the selection error. Its initial values were 0.0205194 and 0.51623, and the final values after 100 generations were 0.00833242 and 0.20453.



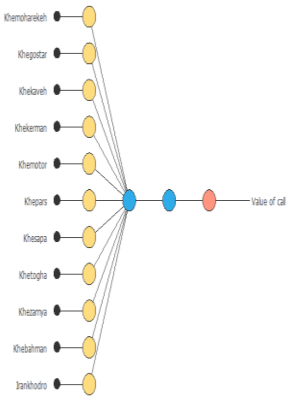


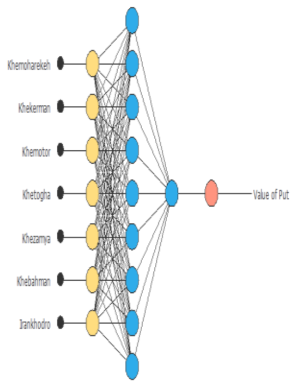
Ga error plot (Call option)

Ga error plot (Put option)

Figure 27. Genetic algorithm error plot

Finally, the figure below displays the selected input variables obtained using GA.





Input selection results using GA(Call option)

Input selection results using GA(Put option)

Figure 28. Input selection results

For a call option, all variables are considered as inputs, whereas for a put option, only seven variables are selected.

Table 23. Input selection results

Criteria	Call option	Put option
Optimal number of inputs	11	7
Optimum training error	0.22713	0.180748
Optimum selection error	0.16625	0.138278

Generations number	100	100
Elapsed time	0:03	0:02

4.9 Artificial Neural Network

After identifying the most significant variables as inputs, the optimal solution can be obtained using ANN. Therefore, the first step was to determine the best network structure.

ID	Architecture	# of Weights	Fitness	Train Error	Validation Error	Test Error	AIC	Correlation	R-Squared	Stop Reason
1	[11-2-1]	27	1.012627	0.972598	0.895377	0.98753	-1274.532153	0.993286	0.986289	All iterations done
2	[11-28-1]	365	1.435926	0.682931	0.727785	0.696415	-683.744339	0.996391	0.992499	All iterations done
3	[11-18-1]	235	1.327969	0.712516	0.793845	0.75303	-933.523759	0.99625	0.992418	All iterations done
4	**[11-11-1]**	**144**	**1.573022**	**0.624059**	**0.663161**	**0.635719**	**-1147.470185**	**0.997034**	**0.994028**	**All iterations don**
5	[11-7-1]	92	1.462255	0.68934	0.670029	0.683875	-1227.492904	0.996504	0.992916	All iterations done
6	[11-15-1]	196	1.431492	0.676107	0.668691	0.698572	-1024.164376	0.996703	0.993298	All iterations done
7	[11-13-1]	170	1.494554	0.613408	0.663761	0.669096	-1099.618861	0.997115	0.994176	All iterations done
8	[11-9-1]	118	1.370727	0.797662	0.767852	0.72954	-1140.318973	0.994629	0.989213	All iterations done
9	[11-12-1]	157	1.496704	0.587755	0.620787	0.668135	-1135.914183	0.997388	0.994759	All iterations done
10	[11-10-1]	131	1.508021	0.629861	0.687006	0.663121	-1171.239625	0.996808	0.993518	All iterations done

ID	Architecture	# of Weights	Fitness	Train Error	Validation Error	Test Error	AIC	Correlation	R-Squared	Stop Reason
1	[11-2-1]	27	0.438008	2.042315	2.172717	2.283065	-1095.741823	0.990789	0.981433	All iterations done
2	[11-28-1]	365	0.607166	1.573988	1.697905	1.646997	-482.515435	0.994808	0.98961	All iterations done
3	[11-18-1]	235	0.638558	1.429636	1.521282	1.566029	-765.697919	0.995107	0.990163	All iterations done
4	[11-11-1]	144	0.580763	1.405848	1.378825	1.721873	-951.741731	0.995557	0.991096	All iterations done
5	[11-24-1]	313	0.611793	1.507854	1.676386	1.63454	-596.860399	0.99494	0.989866	All iterations done
6	**[11-15-1]**	**196**	**0.665671**	**1.23711**	**1.402655**	**1.502244**	**-878.556494**	**0.996655**	**0.993297**	**All iterations done**
7	[11-13-1]	170	0.623052	1.521844	1.649978	1.605002	-880.634679	0.995094	0.990017	All iterations done
8	[11-16-1]	209	0.665209	1.296411	1.474481	1.503288	-841.272597	0.996564	0.993065	All iterations done
9	[11-14-1]	183	0.594596	1.301476	1.519542	1.681814	-892.332813	0.996103	0.992134	All iterations done

Figure 29. Top 5 best network architecture for call option and put option respectively

The blue highlights indicate the top five best networks, whereas the black and bold ones represent the absolute best network. Next, we trained the network by considering the previously mentioned parameters.

Table 24. Training output

Parameters	Call option		Put option	
	Training	**Validation**	**Training**	**Validation**
Absolute error	0.3453	0.4535	0.6371	0.8394
Network error	0.000118	0	0.00011	0
Error improvement	5.69E-19		4.42E-07	
Iteration	71		75	
Training speed, Ite/sec	88.74999		93.7499	
Architecture	[11-5-1]		[11-5-1]	
Training algorithm	Levenberg-Marquardt		Levenberg-Marquardt	
Training stop reason	No error improvement		No error improvement	

Then, a regression analysis (Liviani & Rachman, 2021) and fitting were performed using an ANN. Levenberg-Marquardt is used as an optimization algorithm to prevent getting trapped in local minima.

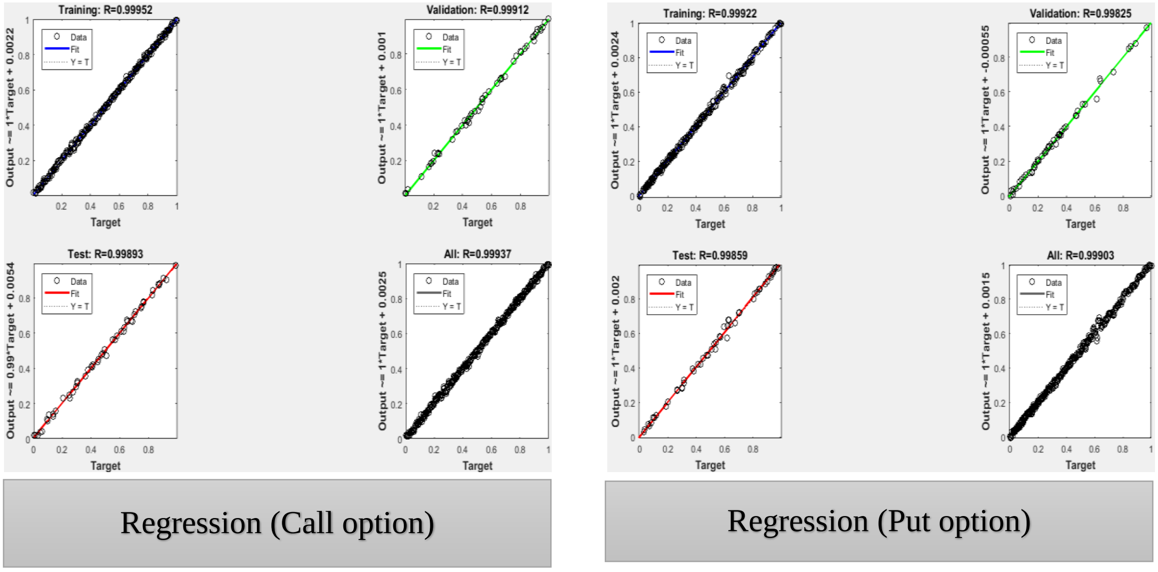


Figure 30. Regression

The above figure shows the regression based on different datasets, including training, validation, and testing. All regressions have R-squared values greater than 99%, which is indicative of an interesting result.

The network performance during each iteration and the best network performance are shown in the figure below:

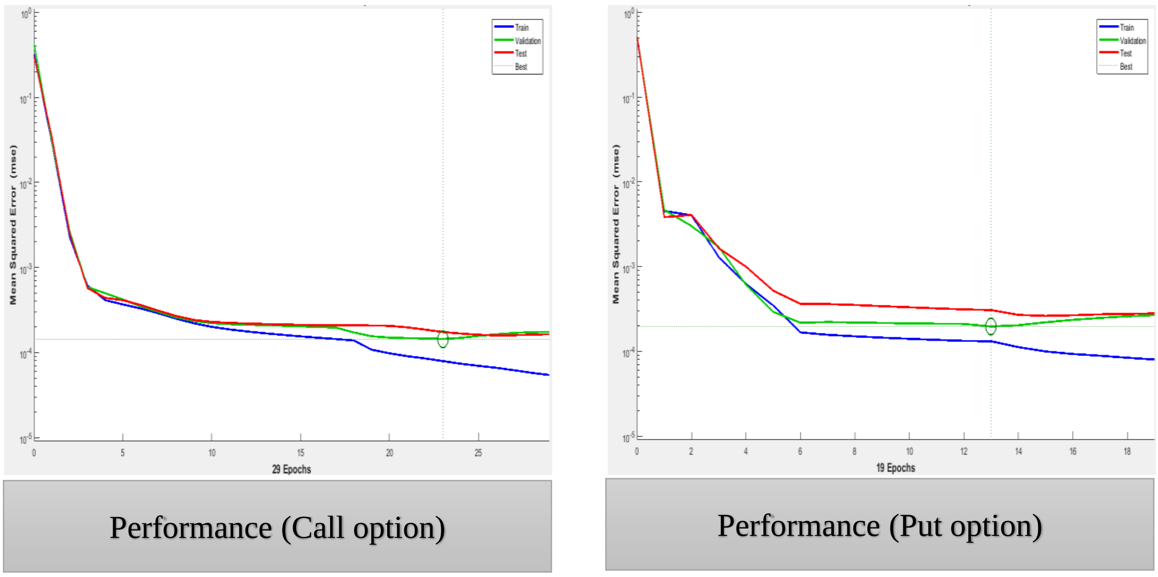
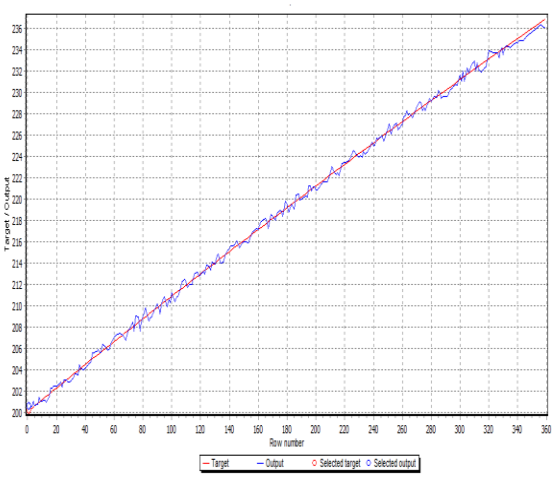


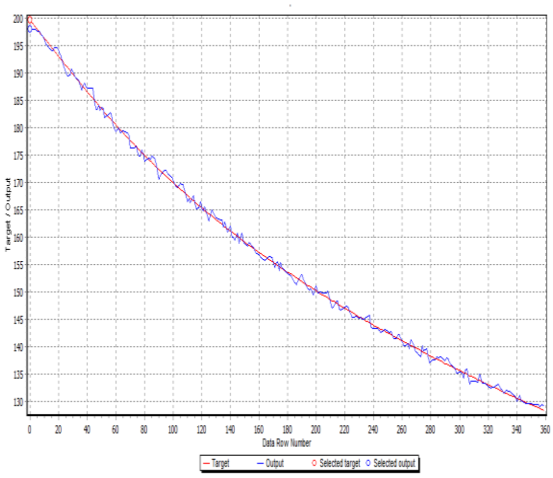
Figure 31. Performance of the network during iterations

The best validation performances for the call option and put option were 0.00014417 and 0.00019657 at epochs 23 and 13, respectively.

The following figure shows the target versus the output. In other words, the actual value is represented by a red line, whereas the predicted value is represented by a blue line.



Performance (Call option)



Performance (Put option)

Figure 32. Performance of the network

Error estimation using various loss functions is presented in the table below.

Table 25. Error tables

Options / Loss functions	Call option			Put option		
	Training	**Selection**	**Testing**	**Training**	**Selection**	**Testing**
Sum squared error	110671	20120.8	25090	301208	54824.5	68779.4
Mean squared error	435.71	379.637	473.39	1185.86	1034.42	1297.72
Root mean squared error	20.874	19.4843	21.758	34.4363	32.1625	36.0239
Normalized squared error	3.8492	3.46104	4.157	2.94547	2.58624	3.23308
Minkowski error	22006	4072.82	4931.6	45101.8	8272.36	10220

In this article, various models such as the Black-Scholes-Merton model, Monte Carlo simulation, Heston model, GARCH model, Lattice model, Jump Diffusion model, Normal Inverse Gaussian-Cox-Ingersoll-Ross model, and Black-Scholes-Artificial Neural Network (BSANN) are used for the prediction of European call and put options. We cannot conclude which model is better because each model has different qualifications and characteristics. For example, the NIG-CIR option pricing model has more parameters. Therefore, this condition can decrease the calculation speed and complicate it. However, the results may be more accurate than the others. An important issue is the accurate and precise setting of the parameters according to the model. To make informed decisions, it is essential to have accurate information about the underlying asset, including factors such as the volatility rate and strike price. Additionally, it is important to be familiar with economic conditions such as the inflation rate and rate of return.

Table 17 shows the results of option pricing using the different models considered. There are also some assumptions: the interest rate (risk-free-rate) is 0.2, the volatility (σ) is 0.42 and the expiration period is one year (360 days). The predicted option prices are obtained for each model with different standard deviations.

Table 26. Different forecasting models

Rank	Model	Strike price	Standard deviation	Option value

		Call	Put	Call	Put	Call	Put
1	**BSANN**	**200**	**600**	**5.43E-05**	**8.00E-05**	**134.63**	**235.21**
2	Heston Option	200	600	0.11	0.11	133.58	232.98
3	BOP	200	600	0.11	1.29	232,96	130.07
4	BSM	200	600	0.11	1.53	236.88	128.27
5	NIG-CIR	200	600	10.62	10.53	131.10	233.17
6	Merton's Jump Diffusion	200	600	13.35	12.58	130.26	232.38
7	Monte Carlo	200	600	16.35	16.58	128.01	237.72
8	GARCH	200	600	84.76	108.44	181.83	286.21

According to the above table, the BSANN model with the lowest standard deviation has the best predictability. However, the GARCH estimation with the highest standard deviation is a relatively unsuitable model for predicting options.

Finally, it is important to consider that the options carry more risks. On the other hand, taking more risk increases potential returns. Thus, before making any investment, it is important to consider your strategy and approach, which includes determining whether you are risk-takers or risk-averse.

5. Conclusion

5.1 Limitation

There are some limitations in different terms, such as the choice of underlying asset. It is important to pay attention to the availability of sufficient data on the number of share transactions and absence of trading halts. Additionally, it is crucial to be careful when selecting an appropriate number of estimated parameters.

5.2 Suggestion

As a recommendation, optimizing artificial intelligence (AI) using metaheuristic algorithms, such as the Chimp Optimization Algorithm (ChOA) and the Aquila Optimization Algorithm (AO), can be effective and applicable in option price forecasting. AI offers several advantages.

1. Speed up calculations
2. Increasing accuracy
3. Uncomplicated

AI methods do not have any assumptions or hypotheses, making them applicable to any problem. They are adaptable to data and complex structures. However, these algorithms can prevent the problem of getting stuck in the local minima or maxima.

Acknowledgment

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